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ON THE ADEQUACY OF THE IDEALIZATION THEORY OF THE POZNAŃ SCHOOL

1. Introduction

The Poznań School methodology of science is in practice the only living philosophical tradition which takes the idealizing nature of scientific theories and laws into account in full detail. Contrary to other traditions in analytical philosophy of science, one of the corner stones of the Poznań School is a detailed theory of idealizations. Related to the concretization of idealized laws, the school also develops a specific theory of the growth of scientific knowledge. Moreover, the Poznań School is notable also for the attempt to synthesize ideas from Marxist philosophy with those in the analytical tradition.

In this paper we consider the adequacy of the concretization schema of idealized laws, proposed by Krajewski (1977) and Nowak (1980, 1989), the leading scholars of the Poznań School. We shall analyze a couple of examples, and introduce a new type of idealizations and concretizations, called *partial idealizations* and *partial concretizations*, neglected in the Poznań School contributions. The partial idealizations and concretizations contribute directly to the Poznań School ideas extending the application area of the Poznań theory of idealizations and making the idealization theory more realistic.

2. The Concretization Schema of Idealizational Scientific Laws

Start with the central idea of Poznań School methodology of science, the concretization schema of idealizational laws (cf. Krajewski 1977, 23—4, Nowak 1980, 119—120 and Nowak 1989, p. 230). Assume that a researcher wants to explain the phenomena *F* in the class of objects *R*. On the basis of the background knowledge some factors, say

$H, p_k, p_{k-1}, ..., p_1$, are assumed to be essential for F. All the factors which are not considered to be essential for F are *reduced away*. Here H denotes the *principal* factor for F and the rest of the factors are the *secondary* factors for F. It is then said that H is the most important factor relative to F and it holds for the secondary factors that if $i>j$, then p_i is more essential for F than p_j, $i, j = 1, ..., k$. For simplicity, it is assumed that there exists only one principal factor for F and that there exist no equi-essential factors for F. These concepts are not defined here, they are used in the paper in the more or less fuzzy sense in which Krajewski (1977) and Nowak (1980) use them.[1]

Using the concepts of the principal and secondary factors the concretization schema of idealized laws is the following:

First, all the secondary factors $p_k, p_{k-1}, ..., p_1$ are *abstracted* away by assuming that $p_i(x) = 0, i = 1, ..., k$ and *assuming counterfactually* that essential, secondary factors p_i (on the given value $p_i(x) = 0$) do not influence on F. Then a simple dependence between F and what is believed to be its principal factor is hypothetically proposed. In this manner an *idealizational law*

(T_k) if $R(x)$ and $p_1(x) = 0, ..., p_{k-1}(x) = 0$ and $p_k(x) = 0$ then $F(x) = f_k(H(x))$

is put forward.

The idealizational law (T_k) or, using Nowak's terminology, *an idealizational statement* (T_k) is concretized with respect to the influence of the secondary factors. The concretization process starts from the most important secondary factor.

Thus, condition $p_k(x) = 0$ is removed and an appropriate correction to the consequence of the statement is introduced. Then the *first concretization* of the idealizational statement (T_k) is of the form:

(T_{k-1}) if $R(x)$ and $p_1(x) = 0, ...$ and $p_{k-1}(x) = 0$ and $p_k(x) \neq 0$ then

$F(x) = f_{k-1}(H(x), p_k(x))$.

Then condition $p_{k-1}(x) = 0$ concerning the most essential secondary factor after p_k is removed, and so on. The *final concretization* relative to the least essential secondary factor p_1 yields a *factual statement* (a fully concretized law):

(T_0) if $R(x)$ and $p_1(x) \neq 0, ...$ and $p_k(x) \neq 0$ then

$F(x) = f_0(H(x), p_k(x), ..., p_1(x))$.[2]

According to Nowak (1980, p. 30) the final concretization of a given idealized law is constructed in practice rather rarely, if at all. Scientists end the concretization process at some point and assume that the influence of the remaining non-concretized factors on the investigated magnitude is "sufficiently small". In other words, the concretization process stops when the real (observed) values of the investigated magnitude are "sufficiently close" to their theoretical values which are yielded by the dependence-equation, only partially concretized.[3]

3. The Adequacy of the Concretization Schema of Idealizational Laws

We shall start with some logical comments. From a logical point of view idealized as well as concretized laws are universal conditional sentences. The original Poznań-idea is to analyze them as material implications. It seems that the presupposed logic should be a *two-sorted, perhaps second-order logic*.[4] But apart from the choice of any adequate logic, there are other, more serious problems connected with the universal quantification over the related idealizing and concretizing conditions.

Consider a set U, the universe of discourse. According to Nowak (1980, pp. 28—31, see also Krajewski 1977, pp. 23—25), $R(x), x \in U$ is a *realistic* assumption which is fulfilled by any element x of the universe of discourse U. Moreover, an assumption concerning the values of a magnitude, say p, is an *idealizing* assumption if and only if the assumption on the values on p are in fact false for *each* object of U, i.e., for each $x \in U$ which satisfies condition $R(x)$. For example

(1) $p(x) = 0$

is an idealizing assumption if and only if 0 denotes the minimum value of magnitude p and it holds for *any* real object x, $R(x)$ that

(2) $p(x) \neq 0$.

Hence, the *complete form* of an idealizing assumption would be a universal sentence (a material implication) of the form

(3) $\forall x: R(x) \rightarrow p(x) = 0$

given that it is known that the consequent part of (3) is false for any x which satisfies condition $R(x)$.

Similarly, removing the consequent of (3) yields a factual statement and the *complete form* of the corresponding realistic assumption with respect to magnitude p then reads

(4) $\forall x: R(x) \rightarrow p(x) \neq 0$.

But there is a bunch of assumptions on the values of magnitude p in U with *different degrees* of idealization (res. with *different degrees* of realisticalness) "between" the two extreme cases (3) and (4). As a simple example consider a set $X = \{x_1, x_2\}$ and a characteristic function $f: X \rightarrow \{0, 1\}$. Here the *only* idealizing assumption in the sense of Poznań School is the following:

(5) $f(x_1) = 0$ and $f(x_2) = 0$,

and the *only* realistic assumption is

(6) $f(x_1) = 1$ and $f(x_2) = 1$,

given that $f(x_1) = 1$ and $f(x_2) = 1$ are the realistic values of f in X.

However, there are the following notable cases as well:

(7) $f(x_1) = 0$ and $f(x_2) = 1$

and

(8) $f(x_1) = 1$ and $f(x_2) = 0$.

In (7) and (8) the assumptions concerning the individuals x_1 and x_2, respectively, are clearly idealizing assumptions concerning a *part* of the universe of discourse X. Returning now to the concretization schema presented in the previous section it immediately follows that *partial idealizations* in the sense of examples (7) and (8) cannot be expressed at all due to the *unrestricted* universal quantification over variables of the principal and secondary factors.

Next consider an example which according to Poznań School is an example of idealization. This example shows that the oversimplified example above is not an empty formal exercise.

Krajewski (1977, pp. 36–8) considers the *idealizational nature* of Kepler's first law in the light of its reduction to classical mechanics. Nowak (1980, p. 72), critisizing Popper, proposes that Kepler's first law:

if the planets do not influence each other gravitationally, then their orbits are ellipses, and the sun is in one of the focal points of the ellipses,

is falsifiable, but only in an indirect way. According to Nowak, it can be subjected to the concretization which is based on lifting the *idealizing assumptions* that the motion of a planet around the sun results from gravitational interaction between these bodies alone.

Neither Krajewski nor Nowak reconstructs this example using the schema of concretization of idealizing laws. And the claim of this paper is that this example is not reconstructable using the concretization schema. Nor is the Newtonian law of general gravitation (in its elementary form) reconstructable using the concretization schema of idealized laws.

Consider first the Newtonian law of gravitation (in its elementary form). It consists of the gravitational force between two isolated bodies; it simply *neglects* the existence of other bodies and other (non-gravitational) forces although they exist in reality. Consider now two alternative ways of reducing the general n-body system to two-body system: Assume first that the masses of the bodies of the system, except the two bodies under consideration, are assumed counterfactually to be zeros. Note, however, that an idealizing assumption in the sense of the Poznań School presupposes that the (idealizing) assumption, that the mass of an object x equals to zero, holds for *every* x in the universe of discourse. Hence, this alternative way to idealize does not fit in the Poznań view, due to *needed restricted* universal quantification.

Assume now that instead of masses of the related bodies, the idealizing assumption says that the gravitational force between two or more bodies is zero, except for the two bodies under consideration. This alternative, however, does not work, because the Poznań view of idealization presupposes that the gravitational force between *any pair* of bodies in the universe of discourse equals to zero. The situation is completely analogous for more complex cases of different combinations of bodies in the universe of discourse. However, to express that the gravitational force between two selected bodies is non-zero and zero for the rest of combinations of bodies, presupposes a restricted universal quantification which does not fit the Poznań view of idealization and concretization.

Consider now the application area of Kepler's first law. Let the universe of discourse, U consist of the sun (s) and the nine planets. The moons of the planets as well as other material bodies are neglected. Assume that we are interested in the orbit of Uranus (u). Then a part of the antecedent of the related idealizing assumptions restricted to the solar system would roughly read as

(9) for any $x \in U$: if $x \neq s$ and $x \neq u$ then $m(x) = 0$

where $m(x)$ denotes the mass of x.

Note that paraphrase (9) is not formalizable as an idealizing assumption in the Poznań sense because of restricted universal quantification. Instead we have here a clear example of *partial idealization*, as introduced in formulas (7) and (8). Neither fits the example in the concretization schema of the Poznań School. Assume that the interference of Neptune (n) with the orbit of Uranus is taken into account. Then the related part of the antecedent would be

(9') for any $x \in U$: if $x \neq s$, $x \neq u$ and $x \neq n$ then $m(x) = 0$

which is not formalizable according to the concretization schema of the Poznań School.[5] This fact is once again due to the needed restriction of universal quantification.

Using gravitational forces instead of masses to formulate the needed idealizing assumptions leads to precisely analogous problems. Nowak (1980, p. 124) reconstructs Newton's second law of motion for inertial systems as the following idealizational statement:

(10) if $O(x)$ and $S(y)$ and x is placed within y and $D(x) = 0$ and $E(y) = 0$, then $F(x) = m(x)a(x)$

where $O(x)$ reads: x is a physical object; $S(y)$ reads: y is a physical system; D are the dimensions of a body; E is the result of external forces; F is the force applied to a given body; m is the mass; and a is the acceleration.

However, it seems that the sentence above is *too idealizing* as a formulation of Newton's second law of motion for inertial systems. The problem here stems from an attempt to define or characterize the predicate $S(y)$ where y is a *physical* system. A natural way to characterize $S(y)$ is to say that $S(y)$ consists of a set of physical objects $O(x_1)$, $O(x_2)$, ..., $O(x_n)$ which are in a definite, regular "order" or "relation" relative to one another. Then $S(y)$ would be equal to some element of the set $\{y \mid y = <x_1, ..., x_n>, x_1, ..., x_n \in O$, O is a set of physical objects and $y \in R$, R characterizes the related regularity$\}$.

Consider now sentence (10). If the characterization of a physical system above or something related is accepted then it follows from (10) that for *any* physical object x: the dimensions of x equal zero. In particular, for any $S(y) = <x_1, ..., x_n>$, $D(x_1) = 0, ..., D(x_n) = 0$ hold. Note that such physical systems are not *real* physical systems. But Newton's second law for inertial systems clearly presupposes that it is also applicable, non-trivially, in systems where $D(x) = 0$ does not hold for every x. However, to guarantee that (10) is so applicable presupposes once again restricted universal quantification of form:

(10) if $O(x)$ and $S(y)$ and x is placed within y and $D(x) = 0$, $x \neq x_i$, x_i occuring in y and $E(y) = 0$, then ...

But a sentence of form (10') is no longer an idealizing statement in the Poznań sense. Moreover, sentence (10) contains some troubles when it is looked at from the point of view of the concretization schema.

Consider the idealizing condition $E(y) = 0$ which says that the result of external forces relative to system y is zero. This idealizing assumption says that y forms a closed physical system, consisting of n bodies ($n = 2, 3, ...$) unaffected by y-external forces. Consider now the concretization schema relative to external forces *E(y)*. In the concretization schema, because of unrestricted universal quantification, the only way to concretize sentence (10) relative to external forces is to replace the idealizing assumption $E(y) = 0$ by condition $E(y) \neq 0$. But this means that the concretization relative to external forces reduces to one "big" step: the influence of external forces to the system *y* is taken into account as one "totality". Consequently it is impossible to present the concretization process stepwise, as the Poznań School requires. In particular, the concretization in the applications of Kepler's law and Newton's law to the solar system goes through several steps. Starting from a simple, idealizing solar system consisting of the sun and one planet, the concretization goes to more complex systems: from a closed two body system to a closed three body system and so on.

4. Notes on the Quasi-Idealizing Assumptions of the Poznań School

It is worth noting that Nowak (1980, pp. 190—4) when he introduces *quasi-idealizing* assumptions, is clearly aware of of the restrictions of the proposed idealization-concretization schema. But let us point out that although quasi-idealizing assumptions resemble partial idealizations introduced above to a certain extent, they are basically different.

As an example of quasi-idealizing assumptions Nowak (1980, p. 190) considers an assumption of the Marxist law of value. This assumption says that the supply and the demand of any given commodity *x* in an economy are in equilibrium, i.e., $S(x) — D(x) = 0$. But according to Nowak, such an assumption is not an idealizing one, because it might very well happen, perhaps very rarely, that the supply and the demand of a commodity really are in equilibrium. Assumptions like that above then generate certain anomalies for the idealization-concretization schema because – according to Nowak – even their existence is excluded (cf. the discussion in Nowak 1980, pp. 190—1). So, Nowak introduces a new type of counterfactual conditions.

Let *U* be the universe of discourse. Condition

(11) $p(x) = 0$

is called a *quasi-idealizing* assumption, if there is a proper subset *K* of the universe of discourse *U* such that

(12) for every $a \in K$: $p(a) = 0$, and

(13) for every $b \in U — K$: $p(b) \neq 0$.

The set *K* is called the *range of realization* of condition (11) whilst the set $U - K$ is called the *range of idealization* of condition (11).

Let us first point out that the Kepler-example above cannot be presented by means of quasi-idealizing conditions. Consider the set of the sun plus the nine planets. It cannot be divided into two parts such that in one part the masses of the objects equal zero, whilst in the other they are non-zero, if the universe of discourse, the set of the sun plus the nine planets, is assumed to consist of real objects.

Second, quasi-idealizing assumptions are not idealizing in the *proper* sense at all. Quasi-idealizing conditions *divide* the *universe of discourse*, a set of *real objects* into two (or in some cases perhaps more) disjoint parts of which some might be empty. But note that they *do not produce ideal objects* from real objects which is the main function of proper idealizing assumptions. In other words, relation $U \cap K \neq 0$ holds, given that $K \neq 0$, in the case where condition $p(x) = 0$ is a quasi-idealizing one, and it has some real content. But if $p(x) = 0$ is a proper idealizing condition, then we *always* get the result $U \cap K = 0$. To put the point slightly differently, if a quasi-idealizing condition has some real content then it generates a partition upon *U*, whereas a proper idealizing condition never does this.

Hence, Nowak's quasi-idealizing assumptions cannot be used to formulate the needed partial idealizations, because idealizations in the proper sense cannot be expressed using quasi-idealizing conditions.

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NOTES

1 Note that neither Krajewski (1977) nor Nowak (1980) define the concepts of one factor being more essential than another, or factors being equi-essential. However, Nowak (1989) defines these concepts exactly, using set-theoretic means. Yet, nothing in our discussion depends on these definitions.

2 Recently Niiniluoto (1986, 1989) has published two important papers on the topic. The original idea of the Poznań School is that the concretization schema as well as the related sentences are formulated as *material implications*. To avoid the undesiderable result that all *idealizational* laws become trivially true, Niiniluoto proposes that material implication should be replaced by an intensional if-then connective.

3 For such cases Nowak (1980) introduces an approximate version of the concretization schema. For comments, see Niiniluoto (1989).

4 Recall that in the related sentences of form $\forall x: R(x) \wedge p_1(x) = 0 \; \ldots \; p_k(x) = 0 \rightarrow F(x) = f_k(H(x))$ (res. with their concretization) $R(x)$ characterizes x as a "material" or some related "actual" or "real" object, i.e., in any case $R(x)$ is not a (real) number, whence $p_i(x)$, $H(x)$ and $F(x)$ are always (real) numbers.

5 Related to paraphrases (9) and (9') consider sentences

(1) $\forall x: A(x) \rightarrow B(x)$

and

(2) $\forall x: A(x) \wedge x \neq a \wedge x \neq b \wedge, \ldots \rightarrow B(x)$.

It is trivially true that a sentence of form (1) implies logically a sentence of form (2), but the converse does not hold.

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