ON THE ADEQUACY OF THE IDEALIZATION THEORY OF THE POZNAŃ SCHOOL

1. Introduction

The Poznań School methodology of science is in practice the only living philosophical tradition which takes the idealizing nature of scientific theories and laws into account in full detail. Contrary to other traditions in analytical philosophy of science, one of the corner-stones of the Poznań School is a detailed theory of idealizations. Related to the concretization of idealized laws, the school also develops a specific theory of the growth of scientific knowledge. Moreover, the Poznań School is notable also for the attempt to synthesize ideas from Marxist philosophy with those in the analytical tradition.

In this paper we consider the adequacy of the concretization schema of idealized laws, proposed by Krajewski (1977) and Nowak (1980, 1989), the leading scholars of the Poznań School. We shall analyze a couple of examples, and introduce a new type of idealizations and concretizations, called partial idealizations and partial concretizations, neglected in the Poznań School contributions. The partial idealizations and concretizations contribute directly to the Poznań School ideas extending the application area of the Poznań theory of idealizations and making the idealization theory more realistic.

2. The Concretization Schema of Idealizational Scientific Laws

Start with the central idea of Poznań School methodology of science, the concretization schema of idealizational laws (cf. Krajewski 1977, 23–4, Nowak 1980, 119—120 and Nowak 1989, p. 230). Assume that a researcher wants to explain the phenomena $F$ in the class of objects $R$. On the basis of the background knowledge some factors, say
$H, p_k, p_{k,1}, ..., p_k,$ are assumed to be essential for $F$. All the factors which are not considered to be essential for $F$ are reduced away. Here $H$ denotes the principal factor for $F$ and the rest of the factors are the secondary factors for $F$. It is then said that $H$ is the most important factor relative to $F$ and it holds for the secondary factors that if $i > j$, then $p_i$ is more essential for $F$ than $p_j$, $i, j = 1, ..., k$. For simplicity, it is assumed that there exists only one principal factor for $F$ and that there exist no equi-essential factors for $F$. These concepts are not defined here, they are used in the paper in the more or less fuzzy sense in which Krajewski (1977) and Nowak (1980) use them.

Using the concepts of the principal and secondary factors the concretization schema of idealized laws is the following:

First, all the secondary factors $p_k, p_{k,1}, ..., p_k$ are abstracted away by assuming that $p_i(x) = 0, i = 1, ..., k$ and assuming counterfactually that essential, secondary factors $p_i$ (on the given value $p_i(x) = 0$) do not influence on $F$. Then a simple dependence between $F$ and what is believed to be its principal factor is hypothetically proposed. In this manner an idealizational law

$(T_k)$ if $R(x)$ and $p_k(x) = 0, ..., p_k(x) = 0$ then $F(x) = f_k(H(x))$

is put forward.

The idealizational law $(T_k)$ or, using Nowak's terminology, an idealizational statement $(T_k)$ is concretized with respect to the influence of the secondary factors. The concretization process starts from the most important secondary factor.

Thus, condition $p_k(x) = 0$ is removed and an appropriate correction to the consequence of the statement is introduced. Then the first concretization of the idealizational statement $(T_k)$ is of the form:

$(T_k)$ if $R(x)$ and $p_k(x) = 0, ..., p_k(x) = 0$ and $p_k(x) \neq 0$ then

$F(x) = f_k(H(x), p_k(x))$.

Then condition $p_k(x) = 0$ concerning the most essential secondary factor after $p_k$ is removed, and so on. The final concretization relative to the least essential secondary factor $p_i$ yields a factual statement (a fully concretized law):

$(T_i)$ if $R(x)$ and $p_i(x) \neq 0, ..., p_i(x) \neq 0$ then

$F(x) = f_i(H(x), p_i(x), ..., p_i(x))$.

According to Nowak (1980, p. 30) the final concretization of a given idealized law is constructed in practice rather rarely, if at all. Scientists end the concretization process at some point and assume that the influence of the remaining non-concretized factors on the investigated magnitude is “sufficiently small”. In other words, the concretization process stops when the real (observed) values of the investigated magnitude are “sufficiently close” to their theoretical values which are yielded by the dependence-equation, only partially concretized.

3. The Adequacy of the Concretization Schema of Idealizational Laws

We shall start with some logical comments. From a logical point of view idealized as well as concretized laws are universal conditionals sentences. The original Poznań-idea is to analyze them as material implications. It seems that the presupposed logic should be a two-sorted, perhaps second-order logic. But apart from the choice of any adequate logic, there are other, more serious problems connected with the universal quantification over the related idealizing and concretizing conditions.

Consider a set $U$, the universe of discourse. According to Nowak (1980, pp. 28—31, see also Krajewski 1977, pp. 23—25), $R(x), x \in U$ is a realistic assumption which is fulfilled by any element $x$ of the universe of discourse $U$. Moreover, an assumption concerning the values of a magnitude, say $p$, is an idealizing assumption if and only if the assumption on the values on $p$ are in fact false for each object of $U$, i.e., for each $x \in U$ which satisfies condition $R(x)$. For example

1. $p(x) = 0$

is an idealizing assumption if and only if $0$ denotes the minimum value of magnitude $p$ and it holds for any real object $x, R(x)$ that

2. $p(x) \neq 0$. 


Hence, the complete form of an idealizing assumption would be a universal sentence (a material implication) of the form

\( \forall x: R(x) \rightarrow p(x) = 0 \)

given that it is known that the consequent part of (3) is false for any \( x \) which satisfies condition \( R(x) \).

Similarly, removing the consequent of (3) yields a factual statement and the complete form of the corresponding realistic assumption with respect to magnitude \( p \) then reads

\( \forall x: R(x) \rightarrow p(x) \neq 0 \).

But there is a bunch of assumptions on the values of magnitude \( p \) in \( U \) with different degrees of idealization (res. with different degrees of realisticalness) "between" the two extreme cases (3) and (4). As a simple example consider a set \( X = \{ x_1, x_2 \} \) and a characteristic function \( f: X \rightarrow \{ 0, 1 \} \). Here the only idealizing assumption in the sense of Poznań School is the following:

(5) \( f(x_1) = 0 \) and \( f(x_2) = 0 \),

and the only realistic assumption is

(6) \( f(x_1) = 1 \) and \( f(x_2) = 1 \),

given that \( f(x_1) = 1 \) and \( f(x_2) = 1 \) are the realistic values of \( f \) in \( X \).

However, there are the following notable cases as well:

(7) \( f(x_1) = 0 \) and \( f(x_2) = 1 \)

and

(8) \( f(x_1) = 1 \) and \( f(x_2) = 0 \).

In (7) and (8) the assumptions concerning the individuals \( x_1 \) and \( x_2 \), respectively, are clearly idealizing assumptions concerning a part of the universe of discourse \( X \). Returning now to the concretization schema presented in the previous section it immediately follows that partial idealizations in the sense of examples (7) and (8) cannot be expressed at all due to the unrestricted universal quantification over variables of the principal and secondary factors.

Next consider an example which according to Poznań School is an example of idealization. This example shows that the oversimplified example above is not an empty formal exercise.

Krajewski (1977, pp. 36–8) considers the idealizational nature of Kepler’s first law in the light of its reduction to classical mechanics. Nowak (1980, p. 72), critizising Popper, proposes that Kepler’s first law:

if the planets do not influence each other gravitationally, then their orbits are ellipses, and the sun is in one of the focal points of the ellipses,

is falsifiable, but only in an indirect way. According to Nowak, it can be subjected to the concretization which is based on lifting the idealizing assumptions that the motion of a planet around the sun results from gravitational interaction between these bodies alone.

Neither Krajewski nor Nowak reconstructs this example using the schema of concretization of idealizing laws. And the claim of this paper is that this example is not reconstructable using the concretization schema. Nor is the Newtonian law of general gravitation (in its elementary form) reconstructable using the concretization schema of idealized laws.

Consider first the Newtonian law of gravitation (in its elementary form). It consists of the gravitational force between two isolated bodies; it simply neglects the existence of other bodies and other (non-gravitational) forces although they exist in reality. Consider now two alternative ways of reducing the general \( n \)-body system to two-body system: Assume first that the masses of the bodies of the system, except the two bodies under consideration, are assumed counterfactually to be zeros. Note, however, that an idealizing assumption in the sense of the Poznań School presupposes that the (idealizing) assumption, that the mass of an object \( x \) equals to zero, holds for every \( x \) in the universe of discourse. Hence, this alternative way to idealize does not fit in the Poznań view, due to needed restricted universal quantification.

Assume now that instead of masses of the related bodies, the idealizing assumption says that the gravitational force between two or more bodies is zero, except for the two bodies under consideration. This alternative, however, does not work, because the Poznań view of idealization presupposes that the gravitational force between any pair of bod-
ies in the universe of discourse equals to zero. The situation is completely analogous
for more complex cases of different combinations of bodies in the universe of dis-
course. However, to express that the gravi-
tational force between two selected bodies
is non-zero and zero for the rest of combi-
nations of bodies, presupposes a restricted
universal quantification which does not fit the
Poznań view of idealization and concretiza-
tion.

Consider now the application area of Ke-
pler's first law. Let the universe of discourse,
\( U \) consist of the sun (s) and the nine plan-
etes. The moons of the planets as well as oth-
er material bodies are neglected. Assume
that we are interested in the orbit of Uranus
\( w \). Then a part of the antecedent of the re-
lated idealizing assumptions restricted to the
solar system would roughly read as

\[
\text{(9) for any } x \in U: \text{if } x \neq s \text{ and } x \neq u \text{ then } m(x) = 0
\]

where \( m(x) \) denotes the mass of \( x \).

Note that paraphrase (9) is not formalizable
as an idealizing assumption in the Poz-
nań sense because of restricted universal
quantification. Instead we have here a clear
example of partial idealization, as introduced
in formulas (7) and (8). Neither fits the ex-
ample in the concretization schema of the
Poznań School. Assume that the interference
of Neptune \( n \) with the orbit of Uranus is
taken into account. Then the related part
of the antecedent would be

\[
\text{(9') for any } x \in U: \text{if } x \neq s, x \neq u \text{ and } x \neq n \text{ then } m(x) = 0
\]

which is not formalizable according to the
concretization schema of the Poznań School.\(^5\) This fact is once again due to the
needed restriction of universal quantification.

Using gravitational forces instead of mas-
tes to formulate the needed idealizing as-
sumptions leads to precisely analogous prob-
lems. Nowak (1980, p. 124) reconstructs
Newton's second law of motion for inertial
systems as the following idealizational state-
ment:

\[
\text{(10) if } O(x) \text{ and } S(y) \text{ and } x \text{ is placed within } y \text{ and } D(x) = 0 \text{ and } E(y) = 0, \text{ then } F(x) = m(x)a(x)
\]

where \( O(x) \) reads: \( x \) is a physical object;
\( S(y) \) reads: \( y \) is a physical system; \( D \) are the
dimensions of a body; \( E \) is the result of ex-
ternal forces; \( F \) is the force applied to a giv-
en body; \( m \) is the mass; and \( a \) is the accel-
eration.

However, it seems that the sentence above is too idealizing as a formulation
of Newton's second law of motion for inertial
systems. The problem here stems from an
attempt to define or characterize the predi-
cate \( S(y) \) where \( y \) is a physical system. A
natural way to characterize \( S(y) \) is to say that
\( S(y) \) consists of a set of physical objects
\( O(x_1), O(x_2), \ldots, O(x_n) \) which are in a defi-
nite, regular "order" or "relation" relative to
one another. Then \( S(y) \) would be equal to
some element of the set \( \{ y \mid y = <x_1, \ldots, x_n> \} \). \( O \) is a set of physical ob-
jects and \( y \in R \), \( R \) characterizes the related
regularity).

Consider now sentence (10). If the char-
acterization of a physical system above or
something related is accepted then it follows
from (10) that for any physical object \( x \): the
dimensions of \( x \) equal zero. In particular, for
any \( S(y) = <x_1, \ldots, x_n> \), \( D(x_i) = 0, \ldots, D(x_n) = 0 \) hold. Note that such physical systems are not real physical systems. But Newton's sec-
ond law for inertial systems clearly presup-
poses that it is also applicable, non-trivially,
in systems where \( D(x) = 0 \) does not hold for
every \( x \). However, to guarantee that (10) is
so applicable presupposes once again re-
stricted universal quantification of form:

\[
\text{(10) if } O(x) \text{ and } S(y) \text{ and } x \text{ is placed within } y \text{ and } D(x) = 0, \text{ then} \quad y \quad \text{and} \quad E(y) = 0, \text{ then} ...
\]

But a sentence of form \((10)'\) is no longer
an idealizing statement in the Poznań sense.
Moreover, sentence (10) contains some trou-
bles when it is looked at from the point of
view of the concretization schema.

Consider the idealizing condition \( E(y) = 0 \)
which says that the result of external forces
relative to system \( y \) is zero. This idealizing
assumption says that \( y \) forms a closed phys-
ical system, consisting of \( n \) bodies \((n = 2, 3,
\ldots)\) unaffected by \( y \)-external forces. Consi-
ider now the concretization schema relative to
external forces \( E(y) \). In the concretization schema, because of unrestricted universal quantification, the only way to concretize sentence (10) relative to external forces is to replace the idealizing assumption \( E(y) = 0 \) by condition \( E(y) \neq 0 \). But this means that the concretization relative to external forces reduces to one “big” step: the influence of external forces to the system \( y \) is taken into account as one “totality”. Consequently it is impossible to present the concretization process stepwise, as the Poznań School requires. In particular, the concretization in the applications of Kepler’s law and Newton’s law to the solar system goes through several steps. Starting from a simple, idealizing solar system consisting of the sun and one planet, the concretization goes to more complex systems: from a closed two body system to a closed three body system and so on.

4. Notes on the Quasi-Idealizing Assumptions of the Poznań School

It is worth noting that Nowak (1980, pp. 190—4) when he introduces quasi-idealizing assumptions, is clearly aware of of the restrictions of the proposed idealization-concretization schema. But let us point out that although quasi-idealizing assumptions resemble partial idealizations introduced above to a certain extent, they are basically different.

As an example of quasi-idealizing assumptions Nowak (1980, p. 190) considers an assumption of the Marxist law of value. This assumption says that the supply and the demand of any given commodity \( x \) in an economy are in equilibrium, i.e., \( S(x) - D(x) = 0 \). But according to Nowak, such an assumption is not an idealizing one, because it might very well happen, perhaps very rarely, that the supply and the demand of a commodity really are in equilibrium. Assumptions like that above then generate certain anomalies for the idealization-concretization schema because—according to Nowak—even their existence is excluded (cf. the discussion in Nowak 1980, pp. 190—1). So, Nowak introduces a new type of counterfactual conditions.

Let \( U \) be the universe of discourse. Condition

\[
(11) \ p(x) = 0
\]

is called a quasi-idealizing assumption, if there is a proper subset \( K \) of the universe of discourse \( U \) such that

\[
(12) \text{ for every } a \in K: p(a) = 0, \text{ and}
\]

\[
(13) \text{ for every } b \in U - K: p(b) \neq 0.
\]

The set \( K \) is called the range of realization of condition (11) whilst the set \( U - K \) is called the range of idealization of condition (11).

Let us first point out that the Kepler-example above cannot be presented by means of quasi-idealizing conditions. Consider the set of the sun plus the nine planets. It cannot be divided into two parts such that in one part the masses of the objects equal zero, whilst in the other they are non-zero, if the universe of discourse, the set of the sun plus the nine planets, is assumed to consist of real objects.

Second, quasi-idealizing assumptions are not idealizing in the proper sense at all. Quasi-idealizing conditions divide the universe of discourse, a set of real objects into two (or in some cases perhaps more) disjoint parts of which some might be empty. But note that they do not produce ideal objects from real objects which is the main function of proper idealizing assumptions. In other words, relation \( U \cap K \neq 0 \) holds, given that \( K \neq 0 \), in the case where condition \( p(x) = 0 \) is a quasi-idealizing one, and it has some real content. But if \( p(x) = 0 \) is a proper idealizing condition, then we always get the result \( U \cap K = 0 \). To put the point slightly differently, if a quasi-idealizing condition has some real content then it generates a partition upon \( U \), whereas a proper idealizing condition never does this.

Hence, Nowak’s quasi-idealizing assumptions cannot be used to formulate the needed partial idealizations, because idealizations in the proper sense cannot be expressed using quasi-idealizing conditions.
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NOTES

1 Note that neither Krajewski (1977) nor Nowak (1980) define the concepts of one factor being more essential than another, or factors being equi-essential. However, Nowak (1989) defines these concepts exactly, using set-theoretic means. Yet, nothing in our discussion depends on these definitions.

2 Recently Niiniluoto (1986, 1989) has published two important papers on the topic. The original idea of the Poznań School is that the concretization schema as well as the related sentences are formulated as material implications. To avoid the undesirably result that all idealizational laws become trivially true, Niiniluoto proposes that material implication should be replaced by an intensional if-then connective.

3 For such cases Nowak (1980) introduces an approximate version of the concretization schema. For comments, see Niiniluoto (1989).

4 Recall that in the related sentences of form $\forall x: R(x) \wedge p_1(x) = 0 \cdots p_n(x) = 0 \rightarrow F(x) = f_1(H(x))$ (res. with their concretization) $R(x)$ characterizes $x$ as a “material” or some related “actual” or “real” object, i.e., in any case $R(x)$ is not a (real) number, whence $p_1(x), \ldots, p_n(x), H(x)$ and $F(x)$ are always (real) numbers.

5 Related to paraphrases (9) and (9′) consider sentences

(1) $\forall x: A(x) \rightarrow B(x)$

and

(2) $\forall x: A(x) \wedge x \neq a \wedge x \neq b \cdots \rightarrow B(x)$.

It is trivially true that a sentence of form (1) implies logically a sentence of form (2), but the converse does not hold.

REFERENCES


Martti Kuokkanen
Department of Philosophy
P.O. Box 24 (Unioninkatu 40 B)
00014 University of Helsinki
Finland

Timo Tuomivaara
Department of Philosophy
P.O. Box 24 (Unioninkatu 40 B)
00014 University of Helsinki
Finland